IMPLEMENTATIONS OF TRACKING MULTIPARAMETRIC PREDICTIVE CONTROLLER

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Abstract: With the recently developed multi-parametric predictive controllers, the potential for widening of the application area of model predictive control (MPC) to low-level control with fast sampling rates has been indicated. However, in order to crate useful industrial controllers, a shift of paradigm from medium-level to low-level control with more emphasis on disturbance rejection is required. This study focuses on the issue of offset-free reference tracking with constrained linear state-space systems that is very important in practical applications. A comparison is made between a setpoint tracking controller and a scheme with a target calculator, both for offset-free tracking in the presence of integrating disturbances. A case study comparison is made on a two-input single-output system of pressure control in a vacuum chamber of a wire annealer.

Keywords: predictive control, parametrization, tracking, feedback systems, state estimation, disturbance rejection

1. INTRODUCTION

Until recently, MPC controllers have been mostly used for control of multivariate processes with relatively slow dynamics, in the mid-layer of control hierarchy (Qin and Badgwell, 2003). While the advantages of their advanced constraints handling capability have also been tempting for use in low-level control as a replacement for PID controllers and similar, such applications were rare due to computing requirements of online optimisation and sampling time limitations. The recently developed multi-parametric (also known as explicit) MPC (mpMPC) approach (Pistikopoulos et al., 2000, Bemporad et al., 2002, Pannocchia et al., 2006) shifts the majority of the required computational effort offline, enabling final controller implementation with very modest computational demand on standard process control equipment, such as inexpensive microcontrollers, programmable logic controllers or even FPGA chips (Johansen et al., 2007, POP, 2003). Due to the parametric explosion of the offline computational demand, the applicability of the mpMPC approach appears to be limited to single-loop controllers or small-scale multivariate systems.

When designing mpMPC controllers for low-level control applications, a paradigm shift from the traditional MPC is required. More emphasis must be placed on disturbance rejection performance, while only a limited subset of features related to control of multivariate systems can be included. In the existing literature on industrial applicability of mpMPC, there appears to be a certain degree of confusion arising from the desire of authors to keep the number of process states as low as possible and to exclude or limit discussion of output feedback issues, resulting in highly application-specific controllers with limited general applicability. The publicly available toolboxes (Kvasnica et al., 2005, Bemporad, 2006) provide only offset-free reference tracking, but not offset-free tracking with integrating disturbances (assumed to be asymptotically non-zero constant: drift, time-varying offsets, etc.).

There are several ways of providing reference tracking for target values of process outputs (y_{ref}), states (x_{ref}), and possibly also inputs (u_{ref}) (POP, 2003, Kvasnica et al., 2005, Bemporad, 2006). In case of true variable reference signals, these signals appear as additional parameters of the resulting multi-parametric controller, with an undesired impact on offline computational demand. This is often achieved by augmenting the process model with additional integrating reference states (Bemporad et al., 2002, Kvasnica et al., 2005), although it may be more convenient to handle references separately from process dynamics (POP, 2003). For fixed target values it is advisable to apply coordinate shifts by using substitutions $y_n = y - y_{reff}$, $x_n = x - x_{reff}$, $u_n = u - u_{reff}$, which do not cause any increase of the number of parameters. Model augmentation to the velocity form, so that the input to the process becomes $\Delta u(k) = u(k) - u(k-1)$, is often used with setpoint tracking to avoid steady-state tracking offset in the absence of disturbances; alternatively, this offset may be removed by choosing appropriate u_{ref} targets. Notice that the velocity form augmentation may also be introduced for the sake of considering actuator rate constraints, and that it may also have a role in integrating disturbance modelling in some implementations of MPC.

A form of integrating control is often required in order to eliminate steady-state offset in the presence of integrating disturbances using concepts of tracking error integration or disturbance estimation (Sakizlis, 2003, Sakizlis et al., 2004). In case of tracking error integration, firstly the model is augmented with the reference state so that the output becomes the tracking error; then, an integrator is appended so that it integrates the tracking error (Bemporad, 2006, Grancharova et al., 2004). This enables integrating control without the use of an estimator, however the scheme is prone to integrator windup in case of unreachable setpoints, and the modification to the cost function may deteriorate tracking performance. Alternatively, with the disturbance estimation approach that is more commonly used in traditional MPC, an observer or estimator is used to estimate integrating disturbance states, appended to the model. In the simplest DMC-style form, an open-loop observer is used for the basic model and disturbance states are estimated at the outputs. In more general formulations (Muske and Badgwell, 2002, Pannocchia and Rawlings, 2003), disturbance states may also appear at the process inputs; typically, a Kalman filter (KF) is used, while practically implementable forms of multi-parametric moving horizon estimators, dual to mpMPC controllers, are still under development (Darby and Nikolaou, 2007).

Considerations of the choice among a MPC controller of unified structure and a target calculator (TC) scheme (Pannocchia and Rawlings, 2003) are also affected in with issues of multi-parametric implementation. In on-line MPC, the dimension of the online optimisation problem of the dynamic controller (DC) in the TC scheme is smaller than the dimension of the unified MPC controller, as the disturbance states are excluded from the DC. In mpMPC, the TC outputs x_{ref} and u_{ref} appear as additional parameters, considerably increasing the offline optimisation problem dimension; however, x_{ref} and u_{ref} are actually a piecewise-affine (PWA) function of the disturbance states and y_{ref} (Sakizlis, 2003). In industrial MPC, the TC scheme is favoured due to its abilities of screening out steady-state infeasibilities and handling online system reconfiguration, although similar functionality can be achieved to a certain degree also with the unified structure by using fixed references x_{reff} and u_{reff} with properly adjusted cost function weights. However, the TC scheme appears to be more suitable for application of infinite horizon costs and other methods that imply control of the system state to the origin.

This paper presents a case study comparison of several tracking schemes on a two-input single-output system of pressure control in a vacuum chamber of a wire annealer described in the next section. Following are sections on simple setpoint tracking, tracking error integration, DMC-style disturbance estimation, and full-featured KF-based disturbance estimation in both unified and TC scheme, discussing their properties and simulation results.

2. PROCESS DESCRIPTION

The control problem is related to the vacuum subsystem of a wire annealing machine that heats the processed metal wire using magneto-focused plasma in an adequate inert gas atmosphere. The task of the controller is to maintain the specified pressure p in the plasma chamber of the annealer that may vary depending on the type of wire and gas. The construction of the vacuum subsystem ensures that a certain pressure profile along the vacuum chamber is maintained to prevent undesired leakage. Vacuum is maintained by several vacuum pumps, connected to different chambers. Rough control of p is performed by adjusting the frequency converters of the pumps connected to chambers at wire exit (right hand side of Figure 1). Additionally, fast regulation of disturbances is carried out by a valve bypassing the sealing before the main chamber, with five times faster response but limited action range. The controller must be able to rapidly suppress fast-acting disturbances that appear during plant operation, such as momentary sealing problems, ignition of plasma, etc. It must also be able to operate over a huge range of operating points, affected by wire diameter, p set-point, machine temperature during start-up, etc. Finally, it must be able to suppress measurement noise efficiently.

For the purpose of evaluation of the tracking schemes, a simple control-relevant linear discrete-time state-space model with the sampling time T_s = 0.2 s. was estimated. It consists of two inputs (pump speed u_1 and valve position u_2) and one output (pressure – the only practically measurable variable in the plasma chamber $p = y$) with simple first order dynamics for each input:

$$A=\begin{bmatrix}0.9244 & 0.0446\\ 0.0446 & 0.6781\end{bmatrix},\quad B=\begin{bmatrix}-0.3863 & 0.0578\\ -0.0678 & -0.3297\end{bmatrix} \quad (1)$$

$$C=\begin{bmatrix}0 & 1\end{bmatrix},\quad D=\begin{bmatrix}0 & 0\end{bmatrix}$$

In discrete time representation, the states have no exact physical meaning.

The controller must consider amplitude and rate constraints: $0 < u_1 < 60$, $0 < u_2 < 10$, $-5\ \mathrm{s}^{-1} < \Delta u_1 < 5\ \mathrm{s}^{-1}$, $-5\ \mathrm{s}^{-1} < \Delta u_2 < 5\ \mathrm{s}^{-1}$. As there is one controlled output and two manipulated inputs, there is a spare degree of freedom, which is available when the narrow u_2 constraints are inactive. It is reasonable to keep u_2 in the centre of its range when possible, so that the controller may effectively react to disturbances in any direction. Ignoring the spare degree of freedom may result in poor coordination of manipulated inputs and awkward control actions.

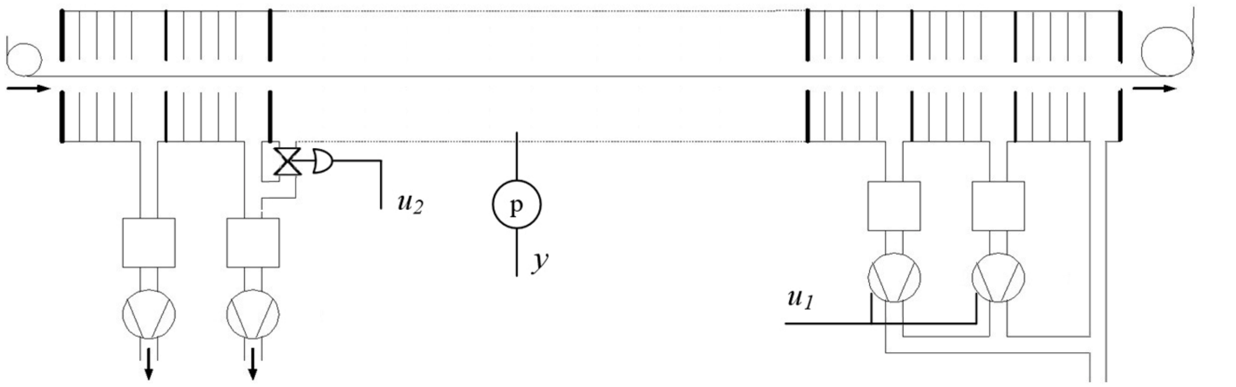

Figure 1. Plasma annealer vacuum subsystem

3. SIMPLE SET-POINT TRACKING

The first step towards a practical tracking controller is offset-free y_{ref} tracking of variable set-points, that is available in various mpMPC realizations (POP, 2003, Kvasnica et al., 2005, Bemporad, 2006). Due to rate constraints, the velocity form augmentation is used. Essentially, this tracking mpMPC controller is still a state-feedback controller. In the absence of state measurements, output feedback can be provided for example by using a Kalman filter (enclosed by default in (Bemporad, 2006)). However, this is not useful in practice for this application because it does not provide integral action and therefore does not eliminate steady-state offset in the presence of integrating disturbances. A modification of the typical tracking implementation is required to address the issue of the spare degree of freedom. A fixed reference $u_{2\mathrm{reff}} = 5$ is imposed to the corresponding additional state $u_2(k\text{–}1)$ of the augmented model, and a relatively small state cost weight $Q_{u(k-1)}$ is added to this state in the cost function. The augmented state includes two original model states $x_1(k)$ and $x_2(k)$, two past inputs $u_1(k\text{–}1)$ and $u_2(k\text{–}1)$ due to the velocity form. Along with the set-point y_{ref}, the total number of mpMPC controller parameters is 5. Figure 2 presents simulation performance of the controller with the following tuning parameters: prediction horizon $N = 7$, control horizon $N_u = 2$, output cost $Q_y = 1$, control move cost $Q_{\Delta u} = [1\ 0;\ 0\ 0.05]$, additional state cost for past input states $Q_{u(k-1)} = [0\ 0;\ 0\ 0.02]$, Kalman filter state noise covariance matrix $Q_w = [0.99\ 0;\ 0\ 0.99]$, measurement noise covariance $Q_v = 0.01$. A controller partition comprising 1599 regions was computed in 133 s on a personal computer P4@3,4GHz. Notice that the number of regions and computation time are influenced by bounds of the parameter space that are negotiable.

For comparison of the tracking approaches, the algorithms are tested with the same y_{ref} set-point sequence of small- and large-amplitude step changes; additionally, an input step disturbance at u_1 with amplitude 4 is inserted at t = 22 s, and an output step disturbance with amplitude 2 is inserted at t = 42 s.

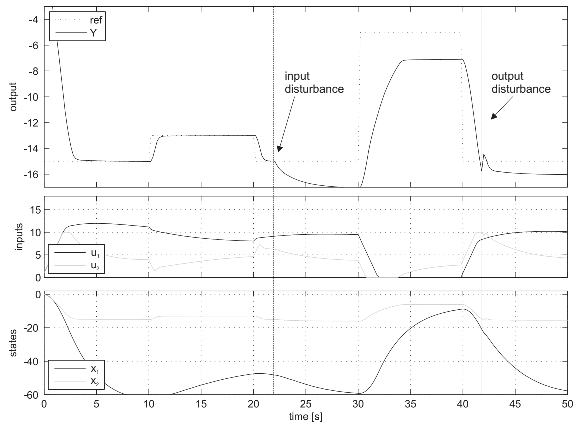

Figure 2. Simple set-point tracking mpMPC simulation with annealer model

4. TRACKING ERROR INTEGRATION

For this approach, in the control scheme the tracking error e_{t} is calculated by subtracting y_{ref} and the output measurement y, and then integrated to obtain the integrated tracking error e_{tI}, which becomes an auxiliary output y_a that the controller attempts to bring to zero. The prediction model is augmented to include the reference state and the tracking error integrator

$$\begin{bmatrix} \mathbf{x}(k+1) \\ e_{\mathrm{tI}}(k+1) \\ y_{\mathrm{ref}}(k+1) \end{bmatrix} = \begin{bmatrix} A & 0 & 0 \\ -T_{\mathrm{S}}C & 1 & T_{\mathrm{S}} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{x}(k) \\ e_{\mathrm{tI}}(k) \\ y_{\mathrm{ref}}(k) \end{bmatrix} + \begin{bmatrix} B \\ -T_{\mathrm{S}}D \\ 0 \end{bmatrix} u(k)$$

$$y_a(k) = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x}(k) \\ e_{\mathrm{tI}}(k) \\ y_{\mathrm{ref}}(k) \end{bmatrix} + 0\,u(k) \qquad (2)$$

The required state estimates are obtained using a nominal model running in parallel to the process in open-loop. Additionally, the velocity form augmentation is applied to the prediction model due to rate constraints, and a fixed reference $u_{2\mathrm{reff}} = 5$ is used for input coordination. The augmented state includes $x_1(k)$ and $x_2(k)$, $e_{tI}(k)$, $y_{ref}(k)$, $u_1(k–1)$ and $u_2(k–1)$. Integrator windup is present when the controller is unable to track the reference closely at step changes (Figure 3, dashed line). For simple windup protection, a constrained integrator with limits 5 and –2 is used for the tracking error (Figure 3, solid line); however, tracking does not work when the integrator is saturated, while windup protection is less efficient if the integrator constraints are relaxed. More complex windup schemes based on the associated unconstrained LQ controller (Grancharova et al., 2004) or reference conditioning are possible. Due to control of an integrated value of the tracking error, the tracking response is considerably deteriorated; this effect may be reduced by penalizing the tracking error e_t in the cost function as a second auxiliary output. In the simulation in Figure 3, the following parameters are used: $N = 7$, $N_u = 2$, $Q_y = 1$, $Q_{\Delta u} = [1\ 0;\ 0\ 0.05]$, $Q_{u(k-1)} = [0\ 0;\ 0\ 0.1]$. A controller partition comprising 1501 regions was computed in 249.90 s.

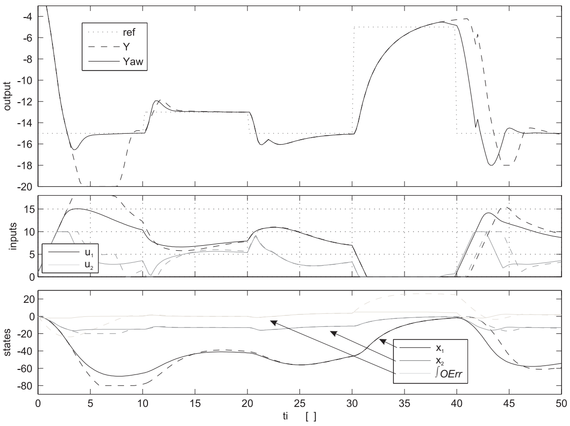

Figure 3. Tracking error integration mpMPC simulation with annealer model. Dashed lines present performance using unbounded integrator.

5. DMC-STYLE OUTPUT DISTURBANCE ESTIMATION (WITH UNIFIED SCHEME)

In this case, integral action is achieved by introducing an integrating disturbance state d added at the process output. A simple disturbance estimator is used: in each step, $d(k)$ is estimated as the difference between the output measurement and the open-loop prediction using the nominal model. The prediction model may be described by the following augmented model

$$\begin{bmatrix} x(k+1) \\ d(k+1) \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} x(k+1) \\ d(k+1) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(k) \tag{3}$$

$$y(k) = \begin{bmatrix} C & 1 \end{bmatrix} \begin{bmatrix} x(k+1) \\ d(k+1) \end{bmatrix} + \begin{bmatrix} D & 0 \end{bmatrix} u(k)$$

In the mpMPC implementation, the augmented state includes: $x_1(k)$, $x_2(k)$, $d(k)$; $u_1(k–1)$ and $u_2(k–1)$ due to rate constraints; additionally, $y_{ref}(k)$ is a parameter of the mp partition (may also be a model state if model augmentation with the reference is used). The advantages of this approach are the absence of integrator windup and that the nominal performance in the absence of disturbances is not affected by integral action. Disturbance estimation may be used either in a unified scheme or with a target calculator. Figure 4 shows the performance with the unified scheme, with the following parameters: $N = 7$, $N_u = 2$, $Q_y = 1$, cost $Q_{\Delta u} = [1\ 0;\ 0\ 0.05]$, $Q_{u(k-1)} = [0\ 0;\ 0\ 0.01]$, $u_{2\text{reff}} = 5$. A controller partition comprising 2560 regions was computed in 379.65 s.

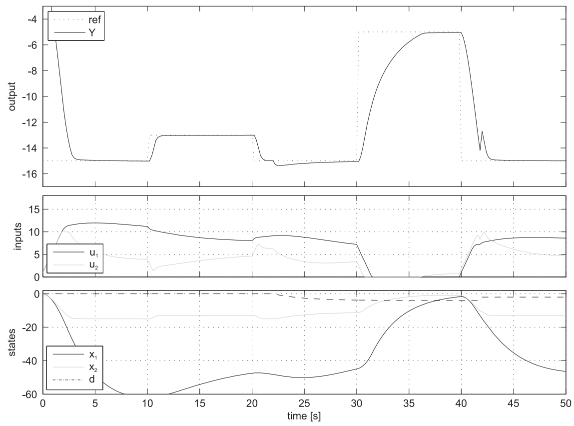

Figure 4. DMC-style output disturbance estimation mpMPC simulation with annealer model

6. KF-BASED DISTURBANCE ESTIMATION WITH UNIFIED SCHEME

With linear models, the scheme from the previous section may be extended with a more capable estimation scheme such as the Kalman filter, by assuming that a stochastic variable $w(k)$ with covariance Q_w is added to the augmented state $[x^T(k)\ d^T(k)]$, and measurement noise $v(k)$ with covariance Q_v acts at the process output. The prediction model and the list of mp partition parameters are similar as in the previous section. Figure 5**Error! Reference source not found.** shows the performance of the scheme with the same controller as in the previous section, with KF state noise covariance matrix $Q_w = [10^{-6}\ 0\ 0;\ 0\ 10^{-6}\ 0,\ 0\ 0\ 1]$ and measurement noise covariance $Q_v = 10^{-3}$.

The KF is much more flexible regarding disturbance rejection performance; notice that the selected tuning parameters result in very similar performance as in Figure 4 with a bit of additional measurement filtering, however the KF covariance matrices were not precisely tuned. The disturbance estimation integrator(s) may be placed at other positions, for example at the process input (Muske and Badgwell, 2002, Pannocchia and Rawlings, 2003).

It should be noted that the KF does not consider constraints and that the robustness of the closed-loop system should be analysed carefully as for constrained systems the separation principle does not hold in general.

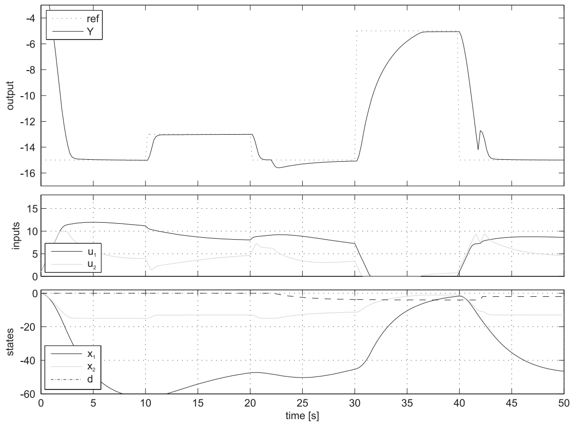

Figure 5. Unified scheme KF-based disturbance estimation mpMPC simulation with annealer model

7. KF-BASED DISTURBANCE ESTIMATION WITH TARGET CALCULATOR

In the TC scheme (Sakizlis, 2003, Sakizlis et al., 2004, Pannocchia and Rawlings, 2003, Pannocchia et al., 2006), the MPC controller is decomposed into the asymptotic steady-state component (TC) and the non-steady-state dynamics (DC). The TC computes the current asymptotic target $\{x^*(k), u^*(k)\}$ from the steady-state component of the model, the reference $y_{\text{ref}}(k)$ and the model and disturbance states $[x^T(k)\ d^T(k)]^T$, using a simplified cost function. The DC drives the dynamic modes to the shifted origin, defined by $\{x^*(k), u^*(k)\}$. An estimator (for example the KF) is used to determine the states of the augmented model with the state vector $[x^T(k)\ d^T(k)]^T$. The controller output is the sum of the TC and DC contributions.

In the mpMPC implementation, the augmented state of the DC includes: $x_1(k)$, $x_2(k)$, $u_1(k–1)$ and $u_2(k–1)$; $x^*_1(k)$ and $x^*_2(k)$ variable references are additional parameters of the partition. The u^* targets are not present; they are replaced with past control signal values with the introduction of the velocity form that facilitates rate constraints, and the Q_u penalty is not used. Notice that the TC is also a mpQP problem of a smaller dimension, with parameters $d(k)$ and $y_{\text{ref}}(k)$, resulting in a piecewise-affine (PWA) partition.

Figure 5 shows the performance of the scheme with the following parameters: $N = 7$, $N_u = 2$, $Q_y = 1$, cost $Q_{\Delta u} = [1\ 0;\ 0\ 0.05]$, $Q_{u(k–1)} = [0\ 0;\ 0\ 1]$, $u_{2\text{reff}} = 5$, $Q_w = [10^{-6}\ 0\ 0;\ 0\ 10^{-6}\ 0,\ 0\ 0\ 1]$, measurement noise covariance $Q_v = 10^{-3}$. A DC controller partition comprising 2320 regions was computed in 269.63 s.

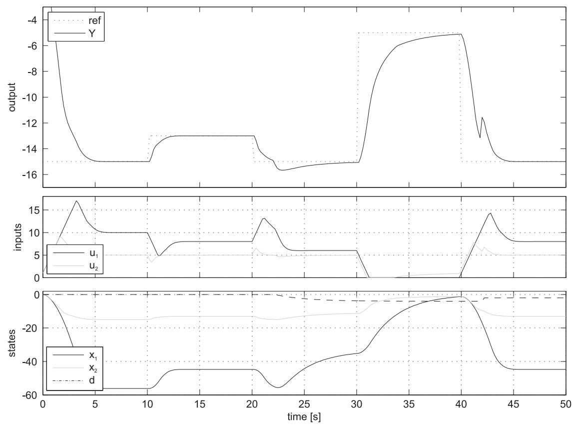

Figure 5. TC scheme KF-based disturbance estimation mpMPC simulation with annealer model

Inconveniently, the number of the parameters of the mpMPC DC is large, as variable references for x^* and u^* targets are introduced – typically larger than the joint dimension of d and y_{ref}. In the DC it is actually possible to reduce the parametric dimension by substituting x^* and u^* as affine functions of d and y_{ref} for each region of the TC PWA partition. However, the calculation of the reduced DC problem is then multiplied by the number of the TC partitions.

Since the x^* and u^* targets are available, the TC scheme is convenient for infinite horizon costs and other methods that assume control of the state to the origin. On the other hand, due to the artificial decomposition of the MPC problem the TC does not consider transient infeasibilities, which arise later in the DC. Back-off from the constraints is used to relieve this issue, and in fact the required back-off may be determined exactly by using soft constraints. However, this requires an iteration of the DC calculation and complicates the control law structure. Notice that the TC may also be used in parallel to the unified scheme for the purpose of performance supervision.

8. CONCLUSION

A variety of tracking schemes can be used in mpMPC. It was shown that the schemes known from online MPC are also applicable in mpMPC. Full-featured schemes are relatively complex and result in large mp dimension that presents a computational burden for off-line computation. Simplifications should be used whenever possible. The Kalman filter may be used for addressing disturbance rejection issues.

The introduction of the target calculator does not decrease the amount of computation as in online MPC. It also does not show an important advantage regarding the ability of system reconfiguration at saturations, as this is also facilitated by the unified scheme. On the other hand, it allows application of advanced control approaches that demand control of the state to the origin.

As there are no output constraints or measured disturbances in this particular application, the practical advantages of MPC over a two-loop PID control scheme are not outstanding. There is a certain improvement in constrained performance compared to PID anti-windup, and better insight into constrained performance is available due to the explicit form of the mpMPC control law. The mpMPC schemes involving an estimator facilitate tuning for efficient disturbance rejection and robustness, which are extremely important in low-level control applications (Gerkšič et al., 2008). However, these topics are outside the scope of this paper.

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